Alg. optim de ridicare la putere

Ex: $a^{50} = (a^{25})^2 = \left(\left(\left((a^2\cdot a)^2\right)^2\right)^2\cdot a\right)^2$

$$a^{50} = (a^{25})^2$$

$$a^{25} = (a^{12})^2\cdot a$$

$$a^{12} = (a^{6})^2$$

$$a^{6} = (a^{3})^2$$

$$a^{3} = (a)^2\cdot a$$

$$a = 1^2\cdot a$$

$$50_{(10)} = 110010$$

$X = 1$

1 $\quad X = X*X*a$

0 $\quad X = X*X$

Calculul optim pt. termenul de indice n din șirul lui Fibonacci

Înmulțire de matrice

$$\begin{pmatrix} a & b \\ c & d \end{pmatrix}\begin{pmatrix} e & f \\ g & h \end{pmatrix} = \begin{pmatrix} ae+bg & af+bh \\ ce+dg & cf+dh \end{pmatrix}$$

$$\begin{pmatrix} e & f \\ g & h \end{pmatrix}\begin{pmatrix} a & b \\ c & d \end{pmatrix} = \begin{pmatrix} ae+cf & be+df \\ ag+ch & bg+dh \end{pmatrix}$$

Fib matr

$$A = \begin{pmatrix} 1 & 1 \\ 1 & 0 \end{pmatrix}$$

1	1	2	3	5	8	13
$n=1$	$n=2$	$n=3$	$n=4$	$n=5$	$n=6$	$n=7$

$$A^2 = \begin{pmatrix} 1 & 1 \\ 1 & 0 \end{pmatrix}\begin{pmatrix} 1 & 1 \\ 1 & 0 \end{pmatrix} = \begin{pmatrix} 2 & 1 \\ 1 & 1 \end{pmatrix}$$

$$A^3 = \begin{pmatrix} 2 & 1 \\ 1 & 1 \end{pmatrix}\begin{pmatrix} 1 & 1 \\ 1 & 0 \end{pmatrix} = \begin{pmatrix} 3 & 2 \\ 2 & 1 \end{pmatrix}$$

$$A^4 = \begin{pmatrix} 3 & 2 \\ 2 & 1 \end{pmatrix}\begin{pmatrix} 1 & 1 \\ 1 & 0 \end{pmatrix} = \begin{pmatrix} 5 & 3 \\ 3 & 2 \end{pmatrix}$$

$$A^5 = \begin{pmatrix} 5 & 3 \\ 3 & 2 \end{pmatrix}\begin{pmatrix} 1 & 1 \\ 1 & 0 \end{pmatrix} = \begin{pmatrix} 8 & 5 \\ 5 & 3 \end{pmatrix}$$

$$A^6 = \begin{pmatrix} 8 & 5 \\ 5 & 3 \end{pmatrix}\begin{pmatrix} 1 & 1 \\ 1 & 0 \end{pmatrix} = \begin{pmatrix} 13 & 8 \\ 8 & 5 \end{pmatrix}$$

<u>Deci</u> A^n ne va da $\begin{pmatrix} t_{n+1} & t_n \\ t_n & t_{n-1} \end{pmatrix}$

Ceea ce ne furnizează urm. alg. de calcul pt.

$\underline{\underline{t_n}}$ (term. al n-lea din șirul lui Fibonacci)

$\longrightarrow$ plecăm de la matr $A = \begin{pmatrix} 1 & 1 \\ 1 & 0 \end{pmatrix}$

$\rightarrow$ calculăm prin alg. optim matricea A^{n-1}

termenul cerut este elem.

de la indicii 1, 1.