

Alg. optim de ridicare la putere

Ex: $a^{50} = (a^{25})^2 = \left(\left(\left((a^2 \cdot a)^2 \right)^2 \right)^2 \cdot a \right)^2$

$$a^{50} = (a^{25})^2$$

$$a^{25} = (a^{12})^2 \cdot a$$

$$a^{12} = (a^6)^2$$

$$a^6 = (a^3)^2$$

$$a^3 = (a^2) \cdot a$$

$$a = 1^2 \cdot a$$

$$50_{(10)} = 110010$$

$$X = 1$$

$$X = X * X * a$$

$$X = X * X$$

Calculul optim pt. termenul de indice n

din şirul lui Fibonacci

Înmulţire de matrice

$$\begin{pmatrix} a & b \\ c & d \end{pmatrix} \begin{pmatrix} e & f \\ g & h \end{pmatrix} = \begin{pmatrix} ae+bg & af+bh \\ ce+dg & cf+dh \end{pmatrix}$$

$$\begin{pmatrix} e & f \\ g & h \end{pmatrix} \begin{pmatrix} a & b \\ c & d \end{pmatrix} = \begin{pmatrix} ae+cf & be+df \\ ag+ch & bg+dh \end{pmatrix}$$

Fib matr

$$A = \begin{pmatrix} 1 & 1 \\ 1 & 0 \end{pmatrix}$$

$$\begin{array}{ccccccc} 1 & 1 & 2 & 3 & 5 & 8 & 13 \\ n=1 & n=2 & n=3 & n=4 & n=5 & n=6 & n=7 \end{array}$$

$$A^2 = \begin{pmatrix} 1 & 1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} 1 & 1 \\ 1 & 0 \end{pmatrix} = \begin{pmatrix} 2 & 1 \\ 1 & 1 \end{pmatrix}$$

$$A^3 = \begin{pmatrix} 2 & 1 \\ 1 & 1 \end{pmatrix} \begin{pmatrix} 1 & 1 \\ 1 & 0 \end{pmatrix} = \begin{pmatrix} 3 & 2 \\ 2 & 1 \end{pmatrix}$$

$$A^4 = \begin{pmatrix} 3 & 2 \\ 2 & 1 \end{pmatrix} \begin{pmatrix} 1 & 1 \\ 1 & 0 \end{pmatrix} = \begin{pmatrix} 5 & 3 \\ 3 & 2 \end{pmatrix}$$

$$A^5 = \begin{pmatrix} 5 & 3 \\ 3 & 2 \end{pmatrix} \begin{pmatrix} 1 & 1 \\ 1 & 0 \end{pmatrix} = \begin{pmatrix} 8 & 5 \\ 5 & 3 \end{pmatrix}$$

$$A^6 = \begin{pmatrix} 8 & 5 \\ 5 & 3 \end{pmatrix} \begin{pmatrix} 1 & 1 \\ 1 & 0 \end{pmatrix} = \begin{pmatrix} 13 & 8 \\ 8 & 5 \end{pmatrix}$$

Deci A^n ne va da $\begin{pmatrix} t_{n+1} & t_n \\ t_n & t_{n-1} \end{pmatrix}$

Ceea ce ne furnizează urm. alg. de calcul pt.

t_n (term. al n-lea din șirul lui Fibonacci)

→ plecăm de la mat $A = \begin{pmatrix} 1 & 1 \\ 1 & 0 \end{pmatrix}$

→ calculăm prin alg. optim matricea A^{n-1}

termenul cerut este elem
de la indicii 1,1.