

# American Computer Science League

2022 Finals • Short Solutions • Junior Division

## 1. Boolean Algebra

$$\begin{aligned} & \overline{(A\overline{B} + B(\overline{A} + \overline{B})) + \overline{A}B} \\ &= \overline{A\overline{B}} \overline{(B(\overline{A} + \overline{B})) + \overline{A} + \overline{B}} \\ &= (\overline{A} + \overline{\overline{B}}) (\overline{B} + (\overline{A} + \overline{B})) + A + \overline{B} \\ &= (\overline{A} + B) (\overline{B} + \overline{A}\overline{B}) + A + \overline{B} \\ &= (\overline{A} + B) (\overline{B} + A\overline{B}) + A + \overline{B} \\ &= (\overline{A} + B) \overline{B} + A + \overline{B} \\ &= A + \overline{B} \end{aligned}$$

D.  $A + \overline{B}$

## 2. Boolean Algebra

$$\overline{A} \overline{(A + \overline{B})} (\overline{A}B) (\overline{A}B) \overline{C} = \overline{A} (\overline{A}\overline{B}) (\overline{A} + \overline{B}) (\overline{A}B\overline{C})$$

To be TRUE all factors need to have a value of 1.

Consider  $\overline{A}B\overline{C}$ . Then  $A = 0, B = 1, C = 0$ .

These values also make the other factors 1, so (0, 1, 0) is the only ordered pair that makes the Boolean expression TRUE.

A. 1

## 3. Bit-String Flicking

$$\begin{aligned} & ((\text{LCIRC-2 (NOT (RSHIFT-1 01100)))}) \text{ OR } \\ & (\text{RCIRC-1 (LSHIFT-2 01011)})) \\ &= ((\text{LCIRC-2 (NOT 00110)}) \text{ OR } (\text{RCIRC-1 01100})) \\ &= ((\text{LCIRC-2 11001}) \text{ OR } 00110) \\ &= 00111 \text{ OR } 00110 \\ &= 00111 \end{aligned}$$

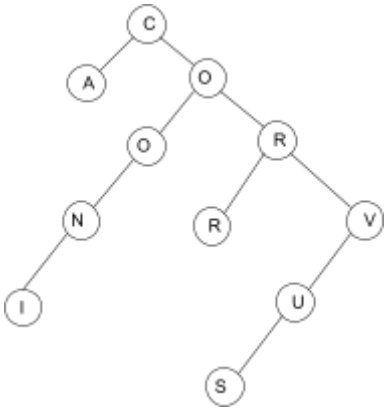
B. 00111

## 4. Bit-String Flicking

$$\begin{aligned} & (\text{NOT (01110 AND NOT 10110) OR (NOT 00110 OR 11011)} \\ & \quad \text{AND NOT (NOT 00100 AND NOT 01010)}) \\ &= (\text{NOT (01110 AND 01001) OR (11001 OR 11011)} \\ & \quad \text{AND NOT (11011 AND 10101)}) \\ &= \text{NOT 01000 OR 11011 AND NOT 10001} \\ &= 10111 \text{ OR } 11011 \text{ AND } 01110 \\ &= 10111 \text{ OR } 01010 \\ &= 11111 \end{aligned}$$

C. 11111

<p><b>5. Recursive Functions</b></p> $f(18) = f(18 - 3) + 4 = f(15) + 4 = 12 + 4 = 16$ $f(15) = f(15 - 3) + 4 = f(12) + 4 = 8 + 4 = 12$ $f(12) = f(12 - 3) + 4 = f(9) + 4 = 4 + 4 = 8$ $f(9) = f(9 - 2) - 3 = f(7) - 3 = 7 - 3 = 4$ $f(7) = f(7 - 2) - 3 = f(5) - 3 = 10 - 3 = 7$ $f(5) = 5 * 2 = 10$	<p>D. 16</p>
<p><b>6. Recursive Functions</b></p> <p>For mod 3 the sequence is:</p> <p>0, 1, 1, 2, 0, 2, 2, 1, 0, 1, 1, 2, 0, ...</p> <p>Therefore, the Pisano period for mod 3 is 8.</p>	<p>B. 8</p>
<p><b>7. Digital Electronics</b></p> $A(\overline{AB} + \overline{BC} + C) = A(\overline{A} + \overline{B} + \overline{B} + \overline{C} + C)$ $= A(\overline{A} + \overline{B} + 1) = A(1) = A.$	<p>A. A</p>
<p><b>8. Digital Electronics</b></p> <p>The direct translation of the digital circuit is: <math>(A + \overline{AB})B</math>. This simplifies as follows:</p> <p><math>(A + \overline{A} + \overline{B})B = (1 + \overline{B})B = (1)B = B</math>. That means that only 2 ordered pairs make it TRUE: (0,1) and (1,1)</p>	<p>B. 2</p>
<p><b>9. Prefix-Infix-Postfix</b></p> $* 2 - / * 8 9 * 3 4 4$ $= * 2 - / (* 8 9) (* 3 4) 4$ $= * 2 - (/ 72 12) 4$ $= * 2 (- 6 4)$ $= * 2 2$ $= 4$	<p>A. 4</p>
<p><b>10. Prefix-Infix-Postfix</b></p> $1 2 \$ 2 7 \$ + 5 /$ $= (1 2 \$) (2 7 \$) + 5 /$ $= (9 81 +) 5 /$ $= 90 5 /$ $= 18$	<p>C. 18</p>

<p><b>11. Computer Number System</b></p> <p><math>2022_{10} = 3746_8</math>  Next year with 3 consecutive octal digits is <math>4012_8 = 2058_{10}</math></p>	<p>B. 2058</p>
<p><b>12. Computer Number Systems</b></p> <p><math>2_8 * 35_8 - 124_8 / 2_8 = (2_{10} * 29_{10}) - (84_{10} / 2_{10})</math>  <math>= 58_{10} - 42_{10}</math>  <math>= 16_{10}</math>  <math>= 20_8</math></p>	<p>D. 20</p>
<p><b>13. Data Structures</b></p> <p>The binary search tree for <b>CORONAVIRUS</b> is as follows:</p>  <p>The depth of the tree is 5 since the depth of the root node is 0.</p>	<p>B. 5</p>
<p><b>14. Data Structures</b></p> <p>The queue is constructed using FIFO as follows:</p> <p>M, MA, A, AR, ARA, RA, A, AT, ATH, TH, THO, HO, HON</p> <p>The next item popped is H.</p>	<p>C. H</p>
<p><b>15. Graph Theory</b></p> <p>There are 2 flights from NYC to Orlando with one stop: NCO and NBO. There are 4 flights from NYC to Orlando with two stops: NPCO, NPBO, NCBO, and NBCO. That makes 6 in all.</p>	<p>C. 6</p>
<p><b>16. Graph Theory</b></p> <p>The number of round trips is the same as the number of cycles in the graph. By inspection, these are: NCN, NPCN, NBCN, BCB, BPB, BPCNB, BPCB and BCNPB. Therefore, there are 8 of them.</p>	<p>D. 8</p>

**17. What Does This Program Do?**

In tracing this program.  $50 > 6$  so  $e = 50 \% 6 = 2$  and using our greatest integer notation,  $f = [50 / 6] = 8$ . Also,  $66 > 7$  so  $g = 66 \% 7 = 3$  and  $h = [66 / 7] = 9$ . Then 2 is a factor of 8 and 3 is a factor of 9 so the output is the value of  $8/2 - 9/3 = 4 - 3 = 1$ .

A. 1

**18. What Does This Program Do?**

This program finds the Fibonacci numbers from the inputs of 1, 1, 2, 3, 5, 8, 13, 21, 34 whose digits add up to the term that they are starting with term 1 = 1 and ending with term 11 = 89. Therefore, term 1 = 0+1, term 5 = 0+5, and term 10 = 5+5 so 3 numbers are output: 1, 5, 55.

A. 3

**19. What Does This Program Do?**

This program swaps values in the original array by comparing adjacent locations once through the loop. It then outputs any value that matches the (index+1) squared. After the first loop, the array changes to the second row of numbers. Then 1, 4, 25, 64, and 81 are in the correct locations.

1	16	4	64	9	25	49	36	81
1	4	16	9	25	49	36	64	81

D. 5

**20. What Does This Program Do?**

In order to find out if a word is a palindrome, the first letter must match the last letter, the second must match the next to last until you get to the middle of the word. If the word is RACECAR, then  $s[0] == s[6]$ ,  $s[1] == s[5]$ , and  $s[2] == s[4]$  which means it is a palindrome. The formula for the subscripts that must match is the one in position  $x$  and the one in position  $\text{len}(s)-x-1$ . The value for **palindrome** becomes 0 (i.e. false) if the characters are not equal to each other.

D.  $s[x] != s[\text{len}(s) - x - 1]$