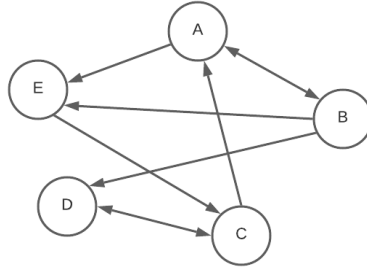


American Computer Science League

2021-2022 • Contest 4: Short Problems Solutions • Junior Division

1. Graph Theory



The directed graph is:

$$\begin{vmatrix} 0 & 1 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 & 1 \\ 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \end{vmatrix}^2 = \begin{vmatrix} 1 & 0 & 1 & 1 & 1 \\ 0 & 1 & 2 & 0 & 1 \\ 0 & 1 & 1 & 0 & 1 \\ 1 & 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 1 & 0 \end{vmatrix}$$

Adding the entries in the squared matrix gives the paths of length 2: 15.

D. 15

2. Graph Theory

There are 4: ABA, ABCA, ADA, and ADCA.

C. 4

3. Digital Electronics

The digital circuit translates to: $A(\overline{AB} + C)$

$$\begin{aligned}
 A(\overline{AB} + C) &= A\overline{A}\overline{B}\overline{C} \\
 &= A(\overline{A} + \overline{B})\overline{C} \\
 &= A\overline{A}\overline{C} + A\overline{B}\overline{C} \\
 &= A\overline{B}\overline{C}
 \end{aligned}$$

This is TRUE for only (1, 0, 0).

B. 1

4. Digital Electronics

The Boolean expression for this circuit is: $\overline{(A + \overline{A} B)} B$

$$\begin{aligned}\overline{(A + \overline{A} B)} B &= \overline{A + \overline{A} B} + \overline{B} \\ &= \overline{A} \overline{\overline{A} B} + \overline{B} \\ &= \overline{A} (A + \overline{B}) + \overline{B} \\ &= \overline{A} A + \overline{A} \overline{B} + \overline{B} \\ &= \overline{A} \overline{B} + \overline{B} \\ &= \overline{B} (\overline{A} + 1) \\ &= \overline{B}\end{aligned}$$

E. \overline{B}

5. What Does This Program Do? (Strings)

The program searches for letters that occur only once in the given string.

Only 4 letters are unique: v, f, m and y

C. 4