

American Computer Science League

2020-2021 • Contest 1: Solutions • Junior Division

1. Computer Number Systems

The binary representation of the decimal numbers from 22 to 32 is as follows:

22: 10110	28: 11100
23: 10111	29: 11101
24: 11000	30: 11110
25: 11001	31: 11111
26: 11010	32: 100000
27: 11011	There are 36 1's.

1. 36 (C)

2. Computer Number Systems

Converting each to binary makes 101100000_2 the largest:

101100000_2	=	101100000_2
535_8	=	101011101_2
$15F_{16}$	=	101011111_2

2. 101100000_2 (A)

3. Recursive Functions

Evaluate each starting at 1:

$$\begin{aligned}f(1) &= 1 \\f(2) &= f(1) + 2 = 3 \\f(3) &= f(2) + 3 = 6 \\f(4) &= f(3) + 4 = 10 \\f(5) &= f(4) + 5 = 15 \\f(6) &= f(5) + 6 = 21 \\f(7) &= f(6) + 7 = 28\end{aligned}$$

Note: the sequence generated is 1, 3, 6, 10, 15, 21, 28, ... These are called triangular numbers - each n th term is the sum of the first n integers. The formula for that sum is $\frac{n(n+1)}{2}$. For $n = 7$, it is $\frac{7*8}{2} = 28$.

3. 28 (B)

4. Recursive Functions

Evaluate the function by working forwards, then backwards.

$$f(20) = f\left(\frac{20}{2}\right) + 1 = f(10) + 1 = 4 + 1 = 5$$

$$f(10) = f\left(\frac{10}{2}\right) + 1 = f(5) + 1 = 3 + 1 = 4$$

$$f(5) = f(5 + 3) - 1 = f(8) - 1 = 4 - 1 = 3$$

$$f(8) = f\left(\frac{8}{2}\right) + 1 = f(4) + 1 = 3 + 1 = 4$$

$$f(4) = f\left(\frac{4}{2}\right) + 1 = f(2) + 1 = 2 + 1 = 3$$

$$f(2) = f\left(\frac{2}{2}\right) + 1 = f(1) + 1 = 1 + 1 = 2$$

$$f(1) = \frac{1 + 1}{2} = \frac{2}{2} = 1$$

4. 5 (D)

5. What Does This Program Do?

The following table can be used to trace the program:

a	b	c
24	2	3
24	6	3
6	6	3
6	8	3
6	8	2

So $a + b / c^2 = 6 + 8 / 2^2 = 6 + 8 / 4 = 6 + 2 = 8$

5. 8 (A)