

American Computer Science League

2023-2024 • Contest 4: Short Problems Solutions • Intermediate Division

1. Graph Theory

The adjacency matrix for the graph is:

$$\begin{bmatrix} 0 & 1 & 1 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 & 0 \end{bmatrix}^2 = \begin{bmatrix} 1 & 0 & 1 & 0 & 1 & 1 \\ 1 & 2 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 2 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 1 & 2 & 0 & 1 & 1 \end{bmatrix}$$

E. 18

Summing the entries in the squared matrix gives the number of paths of length 2 which is 18.

2. Graph Theory

By inspection, there are 19 total cycles, but only 11 originate at vertex A. Remember that ACEA and CEAC are the same cycle so the extra 8 of them don't include vertex A at all. The 11 that include vertex A are:

ABA, AEA, ABEA, ACBA, ACEA, AEBA, ABCEA, ACBEA, ACEBA, ABDCEA, AEDCBA

D. 11

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3. Digital Electronics

This circuit translates to the Boolean expression: $\overline{AB} \oplus \overline{BC}$. The truth table for this is as follows:

A	B	C	\overline{AB}	\overline{BC}	$\overline{AB} \oplus \overline{BC}$
0	0	0	1	0	1
0	0	1	1	0	1
0	1	0	1	1	0
0	1	1	1	0	1
1	0	0	1	0	1
1	0	1	1	0	1
1	1	0	0	1	1
1	1	1	0	0	0

The ordered triples that make this TRUE are: 000, 001, 011, 100, 101, 110

D. 6

4. Digital Electronics

This circuit translates to the Boolean expression: $\overline{\overline{AB} + B + \overline{B} + C}$.

$$= (\overline{AB} * \overline{B}) * (B + C)$$

$$= ((\overline{A} + \overline{B}) * \overline{B}) * (B + C)$$

$$= (\overline{A} * \overline{B} + \overline{B} * \overline{B}) * (B + C)$$

$$= (\overline{B} * (\overline{A} + 1)) * (B + C)$$

$$= (\overline{B} * 1) * (B + C)$$

$$= \overline{B} * (B + C)$$

$$= \overline{B} * B + \overline{B} * C$$

$$= \overline{B}C$$

A. $\overline{B}C$

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5. Assembly Language

This finds which triangular number 78 is. The previous ones are: 1, 3, 6, 10, 15, 21, 28, 36, 45, 55, 66, 78. It's the 12th number in the sequence.

B. 12